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THE ABSTRACT AND CONCRETE IN EDUCATION

THE WORD, THE IMAGE, THE REALITY.

It is coming to be well known, now a days, that to ensure practical and speedy results in the acquisition of a science, be it even that of language, the concrete must go hand in hand with the abstract: the reality or an image thereof with the word expressive of the thing referred to.

The writer's first perception of the truth of this, was, when, a boy at school, he intuitively so to say, made himself a set of solid forms, the simpler ones of wood, the others of paper cut out and stuck or sewed together at the edges, to enable him, the quicker and more surely to take in the teachings of elementary geometry and mathematics: the word or name—a cone, a pyramid, a sphere, a cylinder—conveying the abstract ideal of the form alluded to, while the model itself was the concretion of that idea, its embodiment, its reality.

There are many definitions of the two words; the one due to *abstrahere* to separate, to draw from; the abstraction of a quality inherent in anything from the thing or entity itself; as of a man, a tree, a line, an angle, a figure—the tall, the round, the square, the strong, the right, the curved, the acute, the beautiful in general; or, without reference to any man or thing, or to any person or object, illustrative of the quality set forth—in fact as said before; the mere name or appellation of the thing alluded to; while the concrete (*con* and *creare* to grow together) is the thing itself: that man, that tree, the line A B, the triangle A B C—the abstract proposition and its concrete application to a particular case; the word wind, rain, heat, cold as distinctive from the phenomenon itself or from some tangible, realistic ideal of the thing; as a sensation, a manifestation thereof; or a picture which renders the thing visible: for instance, an inflated sail, an umbrella, a muffled shrinking figure or a perspiring man or animal. The concretizing of an otherwise abstract day of the week and year as "Ash Wednesday," "Good Friday," "Easter Sunday," Dominion, arbor, thanksgiving day—a year: the year 1900, the year of such and such and event—a month: the month of may—a season: autumn, winter, spring, summer—a million: a million dollars, a thousand men—a millionaire: a Gould, a Vanderbilt—a minister of the gospel: a Bonnet, a Beecher, a Parkhurst or a Talmage—a scientist: a Dawson, a Stupart, a Laflamme—a planet: Saturn, Neptune, Mercury—a star: Aldebaron, Sirius, Gamma Cassiopea—a lovely woman: you and you and you and all of you—an ugly woman; no, every thing God made is beautiful of its kind.

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Compare the way in which a child learns the name of a ball, a plate, because he handles it, and has it painted in his eye ; while, not so the days of the week, because they cannot be seen, nor felt, nor a picture be made of them ; and only come to be identified, embodied in the child's mind as Sunday : the day we go to church ; Saturday, the day preceding Sunday, generally a holiday at school ; Wednesday will be the third day after Sunday ; or, more suggestive in the German : the middle of the week. The Queen's Birthday will be remembered on account of its parades and fire works, Christmas because of Santa Claus, Arbor day brings itself back to one's remembrance as the word suggests, and other days to others for their glad or sad remembrances.

The difficulty of illustration consists more in bringing the mode of doing so or the simile to mind, than in applying it. For instance when looking at the driven or driving snow or sleet or rain, you immediately see in it an excellent mode of picturing the wind if not of actually feeling it ; while, with regard to the heat of the sun, a stove, a fire, where there is no apparent motion, it must be felt to render it sensible, or picture to one's self what it is, or its action must be noticed as exemplified in the softening of certain substances, in the liquefying of ice or reducing it to water.

Providence is full of modes or means compensatory of missing or absent faculties, as with the blind, the deaf, the dumb, who are generally endowed with other senses, made more sensitive as that of touch ; the dwarf or hunchback is often times more sprightly both of mind and body ; the ugly and deformed more intelligent, quicker of perception. And if the compensation be not co-existent, it comes in time as where, though woman is the fairer, the prettier, or more handsome while young, the man when old has the advantage of less obliterated features.

In the same way, men not endowed with the mental facilities for the acquisition of the higher mathematics : fluxions and the like, enabling them to deal with and follow up or close upon the varying path and velocity of motion of a jet of water, a stone thrown from a sling or from the hand, a bullet, ball or shell from a rifle, gun or mortar, the motion of a planet or a comet—such men are sometimes endowed by nature with intuitive processes of conception of the phenomenon and of its mode of exposition, and with a facility of illustration which if less satisfactory to the truly scientific, is the more so to humanity in general.

And to proof, again, that the bringing of a thing to mind is sometimes more difficult, than the dealing with it, when thought of, is the fact that so long as, in reading of the proposed Chicago drge. Canal, the writer did not look at the map, he saw nothing of an alarming nature in it, knowing Chicago to be some 300 miles inland or away from our Canadian frontier line along the Lakes ; but so soon as, he one day chanced to look behind him at a map of the

Dominion and United States, it struck him that by the proposed tapping of lake Michigan to the extent of 600,000 cubic ft. of water per minute and running it off towards the Gulf of Mexico through the Des Moines, Illinois and Mississippi rivers; the flow over the Niagara would be lessened by so much and by so much the respective depths of the Great Lakes below Superior, and of the St. Lawrence, below Ontario; though the lakes may be dammed up to their former level, while the draught of the St. Lawrence can not be increased.

This mere accidental glance of his at the map, and which led him to be the first, in December 1894, in calling the attention of our Government to the spoliation, might have occurred to another, but did not and is proof of what is here advanced that the bringing of a thing to mind is more difficult than the doing of it, when thought of; and as a mere idea is or has been so fruitful in other cases; let me here allude to the apparently complicated question of currents and ice drifts about the poles of the Earth and suggest to such as are studying the subject of polar travel and discovery, to take in "centrifugal" force or action, which no one as yet seems to have thought of doing, as one of the important factors not to be neglected in dealing with the subject of arctic and antarctic discovery.

Again in the study of motion in colourless liquids, no man can understand the thing, much less render it evident to others, until the idea occurs to him of coloring the liquids or one of them, whereby its motion may be seen and followed up and submitted to caeculation, as when, with the Gulf Stream, advantage can be taken of its difference of temperature to study it as what it really is, a river in the ocean.

Of professor Tyndall, Herbert Spencer says: "he was endowed with constructive imagination in a high degree. In common with successful investigator, in general, he displayed it in forming true conceptions of physical processes previously misinterpreted or uninterpreted; and again in conceiving modes by which the actual relations of the phenomena could be demonstrated, and again in devising fit appliances to this end. He was an excellent expositor and good exposition implies much constructive imagination."

A prerequisite is the forming of true ideas of the mental states of those who are to be taught, and a further prerequisite is, the imagining of methods by which, beginning with conceptions they possess, there may be built up in their minds, the conceptions they do not possess. Of constructive imagination as displayed in this sphere, men at large—says Spencer—appear to be almost devoid; as witness the absurd systems of teaching which in past times, and largely to this day, have stupefied and still stupefy children, by presenting abstract ideas before they have any concrete conceptions from which they can be drawn. Whether as a lecturer or writer, professor Tyndall carefully avoided this vicious practice.

Huxley in his biographical sketch of Tyndall also alludes to this wealth of experimental illustration, and of his making his way to a position in some respects unique by sheer force of character and intellect, without advantages of education or extraneous aid, to a place in the first rank not only of scientific workers but of writers and speakers.

Now, while a society like the present is not to descend to the roll of imparting knowledge direct to the million, it behoves It, though, to put others in the way of doing so, by being suggestive to them of the ways or at least of some of the modes of so doing in a quick and intelligible manner. Such suggestions may be fruitful and always are in conjuring up in the mind like ways or analogous ones of illustration applicable to the demonstration of other phenomena.

Per adventure we may not have a Faraday, a Huxley, a Tyndall, in the ranks of the society. Every thing is done here by purely mathematical processes, in no way suitable to popularize science of any kind. The writer himself is perhaps the only one who can claim to be of kin to these instructors of the people by his similarity of treatment of what he handles, by the analogy of his modes of illustration, imparting knowledge and suggesting to others how to do it.

Many have wondered and Lord Aberdeen must have been amongst the first to put the question as to why the writer's paper on "*Technical Education in Untechnical language*" of which His Excellency said that it was "*Admirable and most interesting*" and of which the Lord Bishop of Quebec, Mgr Begin and the technologists of the Department of Public Instruction have expressed themselves in such glowing terms, should have failed to secure a place in our vol. of Transaction for 1894, while so much other material of more doubtful utility has been put under cover.

If there was no one there to judge of the merits or utility of the paper, why was it not submitted to an outsider, or even to Lord Aberdeen himself as patron of the society.

Another case, in point, is when some years ago, the writer submitted a paper on "*Hints to Geometers for a new edition of Euclid*" a most important subject for consideration and discussion by the world at large and which if carried out would ere this have been fruitful of a yearly economy of millions of dollars saved to parents in the tuition of their children by the curtailing of the time necessary for the purpose. This paper, it will hardly be credited, was referred for their report to members of the Society, quondam professors at the Seminary of Quebec, totally unacquainted with Euclid, which never crossed the threshold of that Institution and a mere synopsis of the contents of the paper was given by the reporting committee, instead of leaving that for the author to do if the whole paper could not be published, though I believe, of only some eight to ten pages.

The writer therefore after the present session had may be

better decide on leaving the ranks of the Society, as it comes to the same thing whether his usefulness has departed or there be no one in the Society to appreciate it. And yet there is no one to blame for this, except may be the fact that in addition to the four sections—French and English Literature, Geology, Physics, Chemistry and Mathematics, there should or might be a fifth section composed of such men as Huxley, Faraday, Tyndall and the like.

When at the inception of the Society, the writer was made by Lord Lorne and his Counsellors one of its foundation members, it must have been for some supposed speciality — presumably on account of his reduction in his treatise of 1866 in the French language of Euclid's theorems to half their number, while sacrificing none of the conclusions of the Greek Geometer, — or on account of his application of the prismoidal formula to the cubing of almost every known or conceivable elementary form of space-inclosing figure — with the exception only of spindles, their frusta and of ungulae; though even for these the author's system brings out a closer approximation in cask gauging for instance, than any gauging rod can do; since the rod but measures the distance between the bung and head without any reference to the curvature between them, while the prismoidal formula takes in as a factor the half way diameter between the bung and head and reduces the 3 to 4 per cent of error by the rod to a mere fraction of one per cent by the author's process.

Again, may be the author's simplified treatment of spherics was not overlooked by the original board and committee, nor his new and curtailed process of arriving at spherical areas, by the so called spherical excess, tabulated for degrees, minutes, seconds and up to a thousandth of a second or less, in a way to render it possible by mere addition to arrive at the area required of any portion of a sphere, or of a lesser portion of a spheroid in less than 5 minutes.

But likely, the Society since that, absorbed in the more interesting study of Geology, Natural History, Biology, Physics, Chemistry and Electricity, and the writer being one of Sir Edmund Head's, "inferior race" his work was never thereafter looked at or even dreamed of.

And though the author's Alma Mater, the Seminary of Quebec, to cover appearances, made him an M. A. on the strength of it, this institution gave the treatise the cold shoulder because the author was imprudent enough to hint at the possibility of, with such a book, learning mathematics without a master.

The difficulty of illustration, as already set forth, consists not in the doing or how to do, but in conceiving, in fact, of bringing the thing to mind as in illustrating the rotation of the earth upon its axis, when a mounted sphere is not at hand.

For this, an orange or an apple or a well made potato with a bodkin or a knitting needle or a slim pen handle thrust through it

from stem to stern or pole to pole may do good duty; a lamp or light: the sun—a tack stuck in the orange, a point on the earth's surface; and holding the ends of the axis in your fingers or rather between thumb and finger and inclining it to the horizon, while pointing approximately in the direction of the north star or pole of the heavens, walk around the imaginary sun, revolving the orange as you go, upon its axis, thus illustrating day and night, and the seasons and the year, showing the while how the oblique rays spreading over a larger surface temper the atmosphere, rendering it cold or cool or rather not so hot; while when they strike more perpendicularly as at the equator and the tropics, the heat is more intense. Or the double rotation may be pictured by a vehicle or bicycle around a ring, a jet d'eau or the like—the forward motion of the wheel, the yearly—that around its hub or centre, the diurnal; while the smaller or steering wheel may represent the moon, since, though it does not appear to circulate around the earth, it does so in reality by, on account of the difference in velocity of its rotation, presenting itself successively to every portion of the equally rotating Earth.

And in another most striking manner can the rotation of the earth upon itself be made visible, to wit: a rehearsal of Foucault's experiment in the Paris Pantheon but on a smaller scale—suspend an iron or other heavy ball to a swivelled hook in the ceiling over your dining room or office table. Set it oscillating in a direction parallel to one side of the room, to a joint of the floor or table and in less than an hour you will find it oscillating in a plane quite different from that in which you started it. And that the rotation of the suspending string with the house and earth, has had no effect in changing the plane of oscillation, you can see for yourself by holding out your watch at arm's length, its chain between your thumb and finger, and after starting it to oscillate, revolve the chain which will also revolve the watch but have not the least effect in altering its plane of oscillation.

Now the writer does not propose to repeat here his paper on "Technical education of the people, in untechnical language" as read by him in May 1894 before section II of the Society at Ottawa; but merely to give you an idea of how to conceive a mode of illustration of that which you desire to learn or to impart. In these electric days of vast velocities of pulsations or vibrations, by the million millions in a single second of time, of tiny and even microscopic microbes; we are at a loss to see or to believe; but when the gold beater swells out his ½" cube of the precious metal to an area of 20 ft. by 20 or 400 ft. in area, the calculation is elementary that its thickness is reduced to the four millionth of an inch or that it takes or would require 250,000 such thicknesses the one upon the other to make an inch; and if there be 400 leaves in an inch book, you then are forced to the conclusion that, thin as is a sheet of paper, the 400th. of an inch in thickness, yet is each such leaf 600 times the thickness

of the leaf of gold; and that is something you can take between your fingers and feel.

Now as to an appreciation of the fraction of a second; you can call all count to ten in half a second or to twice ten between two successive beats of a seconds pendulum. I do not say to twenty but to twice ten, thus avoiding the two syllable'd words thirteen, fourteen, twenty which would be longer in the saying. If on the Queen's Birthday you stand at one end of a range of 500 men firing a volley and as the sounds will reach your ear successively — sound travelling say 1000 ft. per second—then as the 500 men at say 2 ft. centres, will cover that distance, their firing will to your ear divide the second into 500 distinctly separate intervals. Now go a step further, take up a book of say 500 sheets or leaves to the inch, you can finger it and feel each successive leaf rush past your thumb nail in the tenth part of a second — then have you the second divided into 5000 parts. Again let there be a wheel divided into 1000 parts, each part an inch or so, the circumference therefore say 83½ ft. and the diam. 26½ or thereabout — this is only an ordinary sized wheel, and may revolve once in a second, suppose now the 1000 teeth or cogs or parts are made up each of 1000 thicknesses of paper, then would you have the second divided by this process into a million parts, as each component leaf would pass your eye or abutting thumb or finger in the millionth of a second.

Nor need you be at a loss to understand how the velocity of shot is measured or ascertained, though invisible by the very fact of such tremendous speed on its way. Electricity akin to light, flies around the world at nearly 200,000 miles a second, or for our purposes we may say instantaneously. Then let a screen of tiny wire be put up at say 100 ft. from the gun and another at another hundred ft. further off, so the ball will pass through and strike them both, thus breaking the wire and the electric circuit: the interval of time between these successive breaks gives the velocity of the ball which is as much as 2000 ft. per second more or less according to calibre and charge of powder.

This is enough to guide you and have you understand how the motion of a horse for instance, on the race, may be photo'd 20 times a second, or in 20 different positions of one or a single leap from earth to earth again, and this same process can be pursued with an athlete somersaulting, with a circus man revolving a ball in mid air on his feet, with a juggler throwing and in succession catching balls or knives, etc., and if these photos, or paintings made thereof be placed successively around the inside of a hollow cylinder, with slits in the cylinder above the line of pictures, and if this cylinder be revolved say 8 times in a second, and as the human eye retains the impression of any thing it sees during the 8th part of a second, then thus are you made to see any object in motion you may desire, such as of a man sawing wood, a horse on the gallop, an acrobat's contortions and the like.

The cinematograph which has been the delight and wonder of so many, is no more mysterious than this: it is but the application of the fact that the eye retains the impression of any thing it sees during a fraction of a second. Photos are taken, — say 500 more or less (ten to the second for instance) if each exhibition is to last 50 seconds or a minute — of any object or series of objects in motion. These are reduced in size and painted on a strip of some transparent and pliable or elastic substance as celluloid, capable of being wound and unwound at pleasure. Well it merely requires that the unwinding be at the rate of at least 8 to 10 per second (the more, the more perfect the illusion) and by aid of the so called "magic lantern" projected on to a screen with that velocity to prevent the transition from the one to the other from being noticed, and cause the impression of continuous motion.

Return we now to our theme of the abstract and concrete in education. An exhibit says the "*Scientific American*" which in the educational department under the head of exposition items Chicago attracted much attention from teachers, is the method of teaching mathematics as exploited in the Washington public school at Hackensack, New Jersey, by professor Nelson Haas. The general principles of this system were shown in the New Jersey educational exhibit, the foundation idea being to combine the abstract with the concrete so that the pupil can comprehend in a practical way what he is trying to do. In the primary grades where children from six to eight years old are taught the rudiments of mathematics, the pupil adds to or withdraws from a lot of apples, balls or marbles, nuts, etc., the number to be added or subtracted, so that he has before him a practical demonstration of the problem. In the more advanced grades, pupils are asked to find how many yards of carpeting of given width would be required to cover a floor, or how much plastering to cover the walls, or of paint for the ceiling and similar practical problems.

In each case the room or rather subject of the problem is outlined in a drawing, so as to put before the eyes of the pupil a natural demonstration of what is wanted. The system it appears has proved so efficient that the cadetships in the district of New Jersey, both for Annapolis and West-Point are taken by students who were educated under this system. The same principles are carried out in the higher mathematics on a similar plan, so that pupils from 14 to 18 years of age seem to have a clear comprehension of problems in algebra, trigonometry and even differential calculus; the models clearly and easily exemplifying the differentials of the several orders inquired into.

And there are mental modes of procedure to be resorted to, for such a thing, for example, as computing the weight of bar iron of any scantling; there are certain units to be remembered and they had better be as few as possible, so many things there are now a days to store the mind with. In the same way as we must

know, for very many problems in hydraulics that a cubic foot of water weighs $62\frac{1}{2}$ lbs. and that therefore an imperial gallon is 10 lbs. as there are $6\frac{1}{4}$ gallons to the foot, and that a horse power is equivalent to 330 lbs. raised to a height of 100 ft. in one minute or to 33,000 lbs. raised to the height of one foot in one minute of time, and that thus we can compute the water power for instance of the falls of Montmorency, or Niagara, or of the Hamilton of Labrador, by multiplying the area in feet of cross section of the river above the falls, by the mean velocity of stream, thus getting the number of cubic ft. per minute (the Niagara current being 18,000,000 cubic ft. in that interval of time) and as this number of cubic ft. multiplied into $62\frac{1}{2}$ lbs. the weight of water, and the result into the height of fall gives what we call foot pounds; and as a H. P. is 33,000 ft. lbs., it is therefore evident that dividing the total foot pounds by the No. of them in a H. P. we get the total power of the stream.

Well then a cubic ft. of iron is 480 lbs. remember that and nothing else of what I am going to tell you, for as you shall see, it will take you less time to go through the process mentally than to conjure up your remembrances of other quantities. 480 lbs. divided into twelve slices to the cubic ft. gives you 40 lb. to the inch thick and a ft. square. Then $\frac{1}{2}$ an inch is 20 lbs., $\frac{1}{4}$ = 10 lbs., $\frac{1}{8}$ = 5 lbs., $\frac{1}{16}$ = $2\frac{1}{2}$ lbs., $\frac{1}{32}$ = $1\frac{1}{4}$ lbs., $\frac{1}{64}$ thick gives $\frac{3}{4}$ lb. and so on. Say now you have a bar $3" \times \frac{3}{8}"$ and since $\frac{3}{8}" = 5$ lbs.; $\frac{3}{8} = 15$ lbs. and $3"$ being the quarter of a foot you get the $\frac{1}{4}$ of 15 lbs. or $3\frac{3}{4}$ lbs. Again say you have a $\frac{1}{8}"$ square bar—spread it out mentally into a sheet $\frac{1}{8}"$ thick, it will be 7 times 7 or 49 in width or 6 inches plus $\frac{1}{4}$ of an inch, and the weight will be $2\frac{1}{2}$ lbs., plus $\frac{1}{8}$ of 40 oz. or nearly another oz. together say 2 lbs. 9 oz.

In the *Atlantic Monthly* for March 1894 there is a remarkable article by Nicholas Murray Butler on "The reform of secondary education in the United States." It has come to be distinctly understood says Butler that any far reaching national reform must begin with the secondary schools. Secondary schools have too often sacrificed the necessities of sound training to the local demand for an ambitious programme containing two score or more of school subjects no one of which is pursued far enough or long enough for the pupil to derive from it the educational value it possesses. Or they have erred on the other side and in their devotion to a past ideal, excluded from the curriculum whole fields of knowledge that have grown up within a century or less.

"The consequence is as president Eliot of the National Educational Association, shewed very clearly, several years ago, that the American boy of fifteen or sixteen, no whit inferior to his French, or German fellow in native ability, is from two to three years behind him in acquired knowledge.

"For secondary education and through it for educational organisation, a long step has been taken in this direction by the

proceedings that led up to the appointment of "the Committee of Ten," by the National Educational Association, and by the exceedingly valuable report which that committee has recently laid before the public.

This report is published by the Bureau of Education, Washington, D.C. and to be obtained on request. The members of the committee were: president C. W. Eliot of Harvard University, chairman; Dr. W. T. Harris, commissioner of education; president James B. Angell, of the University of Michigan; president J. W. Taylor, of Vassar College; Mr. John Feltow, of the girls' high school, Boston Mass.; Mr. O. D. Robinson, of the Albany N. Y. High School; president James H. Baker, of the University of Colorado; president R. H. Jesse, of the University of Missouri; Mr. J. C. McKenzie, of the Lawrenceville N. J. School; and professor Henry C. King, of Oberlin College.

Nine conferences were appointed of ten members each; one each on Latin, Greek, English; one on other modern languages; one conference on Physics, Astronomy and Chemistry; one on Natural History, including Biology, Botany, Zoology and Physiology; another on History, Civil Government and Political Economy; the ninth on Geography, Physical Geography, Geology, Meteorology. Among the ninety names will be found many that stand in the foremost ranks of American scholarship, and no one of the ninety was without valuable educational experience of some kind. This fact of itself gives great weight to their recommendations and their exhaustive reports which are appended to the report of the Committee of Ten, are a mine of educational information and suggestion of the utmost value.

Some of the conclusions arrived at may be summarized as: shortening while enriching the school curriculum; the actual abridging of the time now devoted to arithmetic, algebra, geometry, which can be done by merging many propositions into one, making collaries of others, making use of instead of discarding the mechanical idea of motion as in the superposition of figures the one upon the other to prove equalities, and other properties set forth in the demonstrations; that the time now spent on geography in the schools is out of all proportion to the value of the result secured, due to not immediately concretizing or embodying the information obtained and rendering it permanent or lasting by the simultaneous finding or fixing on the map every locality, lake, sea, river, mountain chain or other feature alluded to in the text, and only to the extent required, or so much as memory can take hold of and store for future use; and under this head I would advise that an atlas be always at hand and that whenever reading history, a literary work, a newspaper article, or even a romance, you should pass by not a single name of a place, or feature of the Earth's surface, without stopping to look it up on the map and locate it in your mind in its relative position to other places known to you. Children kick at

this as interruptive of the sequel or thread of the story or article they are at; but, it must be insisted on by the teachers or the parent as the only way of concretizing the information given or acquired by lecture of the article in question; and do not be afraid ladies to interrupt your reading even of a *tête à tête* between two lovers; the lovers will still be there when you return from your atlas, and you will only the more appreciate the situation.

Again the spelling book is disconcerted by the report of the committee and formal grammar is relegated to the subordinate place that it deserves. For two generations, says Bader, the so called grammar school, has conspired with the lower or primary grades to retard the intellectual progress of the pupil in the interest of thoroughness. The arithmetic of many puzzles, the formal grammar and the spelling book with its long lists of child-frightening words have been its weapons. Slowly and with a struggle these are being wrested from it. New knowledge is being introduced to illuminate old, and higher processes to explain the lower.

The conferences on language are a unit in desiring reading aloud in the language to be studied and the writer would add: stopping at each and every word, requiring it, to inculcate its proper and thorough pronunciation and to explain its meaning, as of all the numerous abbreviations which are met with in the reading of a newspaper article for instance, and hosts of words which the pupil has never seen because they seldom or never occur in literature of any kind; a concretization of the word to the ear as its written representation is to the eye. History natural, profane or sacred, geography, catechism or in a novel or narrative of whatever nature.

That laboratory teaching is better than text book teaching which means again that the concrete must go hand in hand with the abstract; as if in discoursing of the pressure of the atmosphere, that fact would not be more easily and quickly credited and better remembered for all time with than without the experimental proof of the phenomenon enunciated.

The conference on History wish to have that subject always associated with the study of geography, and the conference on the latter subject are of that opinion; another phase of the concrete with the abstract and a most important one which any one can put to the test by endeavouring to learn or teach the history of a people with and without the map.

What would the history of a city like London, Paris, Rome or even Halifax amount to in the absence of a map or rather maps of it at the successive periods of its existence, views of its public buildings and other edifices where the history unfolds itself. Look at the wealth of illustration now a days; even the common every day newspaper, teems with portraits of every individual of note alluded to, diagrams explanatory of every field of action; and see what faith we have in graphic illustration of the kind, how invaluable of the

truthfulness thereof, when we do not even stop to ponder on the fact, that in the apparently realistic representation of a battle between the Romans and the Greeks, there could have been no photographers at hand to seize it on the instant, as at the present time we can do. Look at the wealth of illustration in all our story books, and even unto the child's A B C, every letter has its picture, a picture of which the sound of the letter is suggestive as A of age and heigh and hay, B the verb to be and bee and insect, C of to see and sea the ocean and so on of all the other letters of the alphabet and if the letter be a consonant with no homonym, then it is the initial letter of some name or word or thing, some pretty and suggestive embodiment of the letter or of the word and such as to fix it in the memory and cause it to be quickly taken in — another example of the help the concrete is to the abstract in almost everything capable of being illustrated.

Another principle is that each year of the School course, be as far as possible, complete in itself, and self sufficient, and not made dependant on what is to follow. This, say the conference, is essential because thousands of pupils are obliged to leave the High School after one or two years and during that time linguistic, historical, mathematical and scientific subjects should be presented to them in an adequate manner, and more than one foreign language should not be taught at the same time.

Now the writer must thoroughly concur in this opinion and of the importance of its teachings having been witness to the fact during his curriculum of studies at the Quebec Seminary, where the full course runs over eight years; the number of students entering on the course each year — 100 or more — being generally reduced to 20 per cent of the number before they reached the goal. Well? while it is understood that the Laval course is especially adapted rather to professional men and the priesthood, than to the scientist or practical mechanic; this does not do away with the fact that many of those destined in advance by their pretentious parents, to become lawyers, doctors and ministers of the gospel, never — for one reason or another: want of means or otherwise, — get beyond the third or fourth or fifth year, and as mathematics are taught only during the 7th year of the course, while even elementary geometry is not commenced till the 4th year, the consequence is that those leaving after the two or three first years of tuition or even the fourth, go away with a lot of — to them — useless latin, a little greek and a mere inkling of literary attainments, insufficient either for the bar, the pestle or the pulpit, and without even anything in fact to fit them to enter upon the studies of an engineer, an architect, a land surveyor, nor even to start in the world as an apprentice to a carpenter or joiner or intelligent mechanic.

This is due to the fact that geometry is not taught, as it should be, during the very first years of the course; it is put off to

be entered into only "en quatrième" on the fourth or fifth year of the curriculum of studies. This is a most serious, most important matter for consideration and a stop should be put at once to such a system, even if the Legislature or Department of Public Instruction should intervene or interfere in the premises; though it is hard to see how such an Institution can be so blind to the interests of the million in this particular; and seeing that pupils at that age take so much more quickly to anything in the descriptive or constructive line, where they can handle ruler, pencil, dividers, scales and other instruments to their great delight.

This is why education in Lower Canada, the education of the people, is not the practical education it ought to be and especially in Quebec and the districts under its jurisdiction, and which in matters of tuition have in the past and still continue to take their clue from the mother institution of the olden capital, gaining for us as it has at the hands of Sir Edmund Head, one of our governors, the ungracious appellative of "inferior race." The course here is too lengthy by far for our modern requirements; and even in my time, now fifty years ago, when I was but 17 years of age and had then reached what is called "la 2nde," I could not stand it any longer. It was then time for me to enter an apprenticeship, which I did, of Civil Engineering, Architecture, Land surveying and technology in general, studying up at night my mathematics and my physics, chemistry, philosophy, my rhetoric and logic, each in its turn to vary matters and not let them become monotonous, and thus gained fully three years compared to what I would, had I followed in the old rut of conservatism.

Yes, I say, and with the authority of one who knows from long experience: teach mathematics or the elements thereof from the very outset of the course, adding object lessons on the kindergarten system and — not forgetting as I have hinted at before "word lessons" lessons to be learned by reading aloud with some one to correct your pronunciation at each word, and explain the meaning, the why and wherefore of the thing as for instance of the water tight bulkheads or compartments of an ocean liner; words used in political economy, in business, in every day life; words that because the teacher knows them (or may be not) he never has the idea of imparting the meaning of to his young friends the pupils — words of serious import as debtor, creditor, bankruptcy, taxes or assessment, customs duties, excise — territorial divisions and representation in Legislature, Parliament, Senate, Congress, Reichstag, — House of Lords — City wards and municipal representatives of the people — the functions and duties of public officers, civic, civil, judicial, clerical — plaintiff and defendant — circuit, superior, appeal, exchequer and supreme courts and privy council, their functions, prerogatives and jurisdiction, as those of the several ministries, imperial, local, federal, colonial — protection, free trade, licenses, etc.

The subject is so vast, so cosmopolitan that I am wandering from my theme "the abstract and concrete in education" though to be sure; I am still in line, for the very explanation of a word, is an image, a picture, a concretion of it.

It is singular indeed that this necessity of blending the concrete reality with the mere name or abstract ideal of a thing should have been so long in taking root, in asserting itself as a speedy mode of teaching, learning almost every thing conceivable. Where and how did we all learn that this is a table, that a chair, a knife, a ball, a window or a door; but in the very same way that we always have and always will teach children — not by a mere repetition of the word, which to them would be meaningless, but by as often and as constantly pointing to the thing alluded to, in fact taking and handling it, as illustrative of the meaning of the word. Here then is the concrete with the abstract from our very infancy, and what was essential then, why not in after life.

In times gone by there were no other sciences to teach or learn than elementary geometry, mechanics and astronomy — no physics, no chemistry, no acoustics, no optics, no pneumatics, no electricity, no photography, one could then afford the time to dwell on speculative propositions; but life is too short for that in this 19th century of ours, and no time can be given to a merely theoretical proposition, or that has nothing else to recommend it than the amusement it affords. No more time is now wasted on the geometrical solution of the squaring of the circle, the trisection of an angle or the duplication of the cube, since there are so many other and quicker modes of arriving at them with all the accuracy required.

It becomes absurd or useless now a day to say that pure geometry repudiates the mechanical idea of motion, since Euclid himself could not get on without it, as evidenced in one of the first propositions of the first book where he puts one figure on another to demonstrate their equality, and since he did it once, why not again and again which would have enabled him to simplify his demonstrations and reduce the number of his propositions. See for instance the long and round about way of his showing how the angles at the base of an isosceles triangle are equal, while, by supposing the angle at the summit bisected and folding one half of the figure over on the other a very few words suffice to show the truth of the enunciation. Again Euclid makes four separate propositions of what can be expressed in one, to wit that: parallelograms and triangles upon the same or equal bases and between the same parallels are equivalent in area; by merely supposing the equal bases superimposed and remembering that every triangle being the half of its corresponding parallelogram, and that what is true of the wholes is true of the halves, therefore must the truth of the four separate enunciations become apparent. And how much more short and facile would not have been, Euclid's whole theory of parallels and of alternate, exte-

rior and interior angles, had he only called attention to the fact or made a corollary of his definitions to the effect that equal inclination of a line to others gives or implies equal angles.

There is not, there can not be the same interest manifested or even felt by any one in the study of a theorem when he does not see just of what use such theorem may be, where it can have its application; but the moment the student sees the trend of the subject under consideration, he immediately takes to it and more especially so if it speed him on his professional work. Take for instance the theorem that "the tangent is a mean proportional between the whole secant and the portion thereof which lies without the circle. This the engineer would look upon as a mere exercise in geometry and likely not trouble himself about its demonstration, if not told how it applies in the solution of the problem of running a railway curve through two points or cities in such a way as to become tangent to and connect with or branch into an existing railway. Here again then you see that while the theorem is the abstract of the thing or proposition, its application to a case in practice, is its concrecion, its embodiment into real life.

Why also is any interest manifested in the demonstration of the fact that the parts of two secants without the circle are reciprocally proportional to the entire secants? Simply because it is instrumental in the solution of the case in trigonometry where the three sides are given to find the angles. See the interest that centres in what is called the square of the hypothenuse, the so called *pompéian*, the 47th of the first, because of its application to the solution of so many cases in practical engineering, architecture and land surveying, as in the erection of one line perpendicular to another; finding for instance what length of ladder it takes to reach to a certain height on a house or wall, with its foot at a given distance therefrom; how when in a semicircle it shows the perpendicular to the diameter from its apex, to be a mean proportional between the segments of the diameter or base, and how by the use of this fact, it suffices, to get at the radius of a railroad or other curve, to measure any cord of it, and the versed sine or distance at the centre between the cord and arc, then squaring half the cord, dividing by the known portion of the diam. to get the unknown segment which added and the sum halved bring out the sought for radius; for I need hardly remind you one and all that, curves on railways are of such magnitude of radius as to preclude the possibility of laying them out, as a gardener would a flower plot by means of a rope or chain attached to a picket at the centre; though to put it down on paper on a small scale the radius must be known.

Again, what more than mere curiosity would be manifested at the fundamental principle of all geometry, that the three angles of a triangle are together equal to two right angles, were it not that the practical application of the knowledge of this fact enables one,

not only to measure inaccessible distances on the earth's surface, but also the distances to the sun and moon and planets, culminating in our knowledge of latitudes and longitudes and of the figure of the earth and how to find our way about it and trace out the trajectory of a comet on the celestial concave, with the same facility the engineer makes proof of in running a straight or curved road amidst the forests and waters of the earth.

And with other curves than the circle, as the parabola; there is nothing so beautiful as to entrance one at its mere appearance or on considering its properties in the abstract; but how all this warms into admiration, when we find that a study of its form permits us with unerring accuracy to throw a shell into the fortress of the enemy—to calculate how high, how far a jet of water from a reservoir, from a fire hose will reach; and when applied to a speaking trumpet will concentrate the voice and allow it to be projected to a distance; or the curve suited to a reflector which in a light house gathers the rays of light and sends them off together on their errand of humanity.

And on a smaller scale, a more modest and elementary: if you would interest the pupil in the describing of a circle in a triangle, tell him how this will or may be some day utilized by him in his trade of tin or copper smith or plumber, in cutting from a triangular piece of metal the biggest bottom of an oyster can or water pail, the piece may be susceptible of; or a milliner the crown of a hat or bonnet from a three cornered piece of paste board, or the bottom of a basket.

No adequate idea can be formed of the difference in the time of learning a thing by doing it oneself instead of merely being told how to do it. No doubt the mere looking at the figure of what is to be done as, to describe a triangle of which the three sides are given, to bisect an angle, to erect or let fall a perpendicular, the following up the process with eye intent is a step toward the full understanding of the thing to be done; but how much more thoroughly is the thing not understood, when you put your hands to it, your own hands; as had you in joinery to so match two boards as to make a perfect joint, no amount of telling, nothing but an oft repeated trial at the thing could render you successful in its accomplishment.

This system the writer during his long career in the education of his fellows has always carried out. Having for years had as many as a dozen students in his office, in preparation as Technologists, in Engineering, Architecture, Land Surveying, every one of them was made to do the thing required. It did not suffice for one of them to look on while another held the ruler, another the dividers and a third the pencil; no! each of them had to perform the operation. In this way three months sufficed for the whole teaching of plane and spherical geometry and trigonometry, the mensuration of all areas and the

application of the prismoidal formula for solid contents to every imaginable elementary solid as illustrated on a board of some two hundred models, of which the writer, had the honor of entertaining the Literary and Historical Society Quebec in 1872 or some 25 years ago, under the heading "Geometry and the Stereometrical."

The solid figures now used everywhere and considered essential to a prompt understanding of how to measure their areas and solidities, and though they may themselves be looked upon in the abstract as similar figures of any other size, have their concretions or embodiments: the one, the middle frustum of a spindle, as a cask of any size or curvature—the frustum of a cone on its larger base, a salting tub or when reversed a butter firkin, a waste paper basket, a tumbler or drinking goblet, a brewery vat, a bucket—its prisms, the forms of solids of construction—its prismoids, roofs of every kind, bins and other vessels of capacity, railway embankments and excavations,—cones, pyramids and their frusta, the sphere and spheroids and their sections; illustrative for instance of the facility, with the one quarter of the hemisphere at hand and its tri-rectangular triangle, of how one can see the way in which a spherical triangle may become so small as to be equivalent in area to a plane triangle, and so great as to equal in its surface one half of that of the sphere itself, with the sum of its angles formed by the three pairs of component planes, each pair spreading or opening till the contained angle reaches the limit of two right angles, finally becomes equal to six right angles, and illustrative of what in spherics is termed spherical excess and of its use in working out the true area of any extent of country on the surface of the earth.

One cannot too strongly insist on the fact that nothing to the young pupil is so instructive because so amusing, as any solid form he can lay his hands on and turn over on all sides and manipulate, and asked or told what it is representative of, according as it stands on its broader or its lesser base or on its apex; as when a hemisphere suggests a dome when lying on its base, while if reversed it instantly becomes a cauldron; a right angled prism, not only the image of a block of buildings but of each and every of its component stones or bricks, or other parts; an elongated prismoid, a stick of square or wancy timber, a vessel of capacity—turned upside down, a roof, a railway cutting—an embankment when reversed; an elongated frustum of a cone or may be I should rather say the frustum of an elongated cone, the plug of a stop cock, the shaft of the Grecian Doric column; the prolate or elongated spindle, the weaver's shuttle,—its half, cut lengthwise, a boat, a skiff or canoe—and with bottom cut away and ends cut short, a scow or flat;—or, the half spindle when cut the other way or across its longer axis, a filtering bag if held as such, a minaret if turned point upwards; the same spindle with both its ends cut short, the windlass of a ship or the shaft of the connecting rod of a steam engine, or when unequally cut, the shaft of a grecian

or a roman column between base and capital; the acute ungula of a sphere, the image of one of the component sections or ungulae of the orange which when peeled off reveals to the astonished and admiring eye the very poetry of geometrical conception.

Yes the orange or musk melon with its beautifully ribbed exterior must be among the models either in its reality or in image as suggestive to the pupil of the meridian lines which on the globes he studies parcel out the earth and heavens into groupings of land and water and the constellations, and when other lines are drawn across these equidistant from the poles and the equator, parallels of latitude so called, circles of declination, crossing the meridians at right angles and the intersection of the two, indicative of the exact position of any point upon the surface.

Again there is the quarter-sphere remindful of the vaulted apsis of a church, the head of a nich; prismoids illustrative of a square based tower, roof or other structure terminated by an octagon at top, or by a square turned angle-wise to the other or by a circle; the wedge illustrative of the implement of that name or when erect, of a ridge-pole-camping tent or hut; the twisted prism or parallelo-pipedon to cut a hand rail from for the well hole of a staircase; the curved cone and twisted as of a horned animal or powder-flask or cornucopia, etc.

There are also the compound solids or which may be decomposed into a cylinder and hemisphere or other frustum of a sphere illustrative of a mortar or engine of war for throwing shells—the hemisphere and frustum of an elongated cone: a gun, a cone and frustum of a sphere, a buoy — or the gas buoy composed of the best portion of a sphere or spheroid (that is more than half) surmounted by a hollow or concave cone or the frustum of one and illustrative again of a turkish or a moorish dome or finial or minaret; but these few compound figures are merely suggestive of how others may be grouped or applied the one to the other: the cone to the cylinder or to the frustum of a cone as the image of a tower with vertical or battered sides and a conical roof, or of a hay stack.

The sections or frusta of the cones and conoids, straight and curved give the ungula or hoof, the lunette headed opening intersecting a curved or vaulted ceiling or a dome, while the plane sections illustrate the triangle, ellipse, circle, parabola, hyperbola.

Now, there are compound solids illustrative of the lines of penetration or of intersection thereof; or the frusta of cones and cylinders and spheres can be put together or held by the pupil in juxta position in a way to show the projections of said lines of penetration as looked at laterally or endwise or from above: as of two cylinders representative of intersecting vaults or sewers or drainage pipes, water or other conduits which when of equal diameter, give

straight lines of horizontal projection; and when curved more or less are more or less interpenetrating as the diameters are more or less unequal.

But where the curiosity, the interest, the emulation of the pupil comes in, is in the study of the development of surfaces; and in verity nothing can be imagined so instructive to them as this, so suggestive, so educative of his apprenticeship of future tradesmanship, either as tin or copper smith, or plumber, cooper, joiner, tailor, milliner, dress maker and what-not; for as the tin and copper smith and plumber, must know what shape to cut his metal to, to form an elbow, so must the artist charged with robing the human form divine be aware of the peculiarly doubly curved or concavo-convex outline which a sleeve must bear to fit it to the body at the shoulder, and so of other contours, and the milliner to make a hat or bonnet must be able to describe the circular or elliptic crown, as well the more or less splayed lateral environment and finally the rim which, all, must fit the one the other and should do so if artistically shaped, without the help of scissors to bring them into contact.

Let then the boy or girl or pupil or apprentice of the "Kindergarten" be taught to take a cone for instance, and applying to it a piece of paper in a way to have an edge of it run straight from the apex to the base, environ it, envelope it, invest it, wrap it if you like, close fittingly and then cut off the surplus paper with a scissors or sharp cutting knife. Now let him remove the envelope and spread it out on the table or a plane faced board, and he will have the image of the sector of a circle. Then let it again be put together or its edges be made to meet and envelope, if allowance has been therefor made in the cutting; and let the edge be stitched, or glued, or pasted, and you have the image of the cone, of the extinguisher, of the cornet, of the spire of a belfry. The smith sees and notes this and makes unto himself a pattern or patterns of that developed shape which when its edges are made to meet, will reproduce the cone.

And now for the developed lateral surface of the frustum of a cone. Wrap as before and then unwrap the solid, and spread out the envelope on a table and you will have a section or portion of the annulus of a circle, which when its straight or lateral edges are again brought together will be suggestive to you of the lamp shade, the lateral portion of a pail, a pie dish, or a goblet, of the ordinary milk pail of our farmers, of the surrounding of a hat or bonnet, of a collar or splayed sleeve, cuff, etc., and in the same way as, to draw the curved portion or base of the sector to form a cone, you must take for centre of the curve or circle the summit or apex of the cone which thus makes known the radius required; in the same way, to draw the lateral envelope of the frustum of a cone, the splay being given or assumed and the diameter; you must prolong or produce the sides to meet in a point which will be the apex of the cone of which the frustum forms a portion, and the

concentric upper and lower or inner and outer curves drawn from this point, with enough to spare for overlap at joint, will solve the problem.

And if the lateral portion of a ladies hat, I mean that between the crown and brim is, to give it a jaunty look, to be higher on the one side than on the other, the milliner will not be slow at seeing or if she does not, must be told of it, that in such case the upper and lower lines of the annulus being non parallel and therefore non concentric, they must each of them be described from different points, or centres the further apart, the greater the difference of the height or breadth of rim of one side as compared with the other.

Elbows can of course be also traced out and at what ever angle : right, acute or obtuse, by simple rules though too lengthy to dwell on in a paper which does not purport to teach the mode of doing, but is merely suggestive, illustrative to the teacher of the value of models for the purpose, and of the extent to which the concrete forms or realizations of the abstract ideas of certain shapes, quicken and facilitate the intelligence thereof and the *modus operandi*.

But the prettiest solution, most difficult of conception had not nature itself led the way by suggesting it in the orange and ribbed structure of the melon, most difficult of execution, and where the concrete form is most helpful to the abstract idea of how the thing is to be done, is that of the developed surface of a sphere or spheroid as for the covering of a ball to play with, or the formation of one for geographical or uranographical purposes, or the ball of a church steeple, or of an observatory, etc., or of any portion thereof as for a dome or vaulted ceiling, and the only and best model for this is the skin or peel of the orange, which can be divided along its equator into equal parts the more the better, for a more perfect development of the double curvature—with lines drawn from the poles or opposite ends of the stem or imaginary axis. These lines being cut through, except at the equator, the skin or peel can with the help of the curved bowl or handle of a tea spoon be removed without a flaw and the whole spread out upon a plane, and exactly illustrative of how to proceed in shaping the semblance of a ball, a dome or a balloon.

Yes, it may be averred, without the fear of contradiction that this concretizing of the abstract, reduces in a ten fold manner the study of geometry and trigonometry both plain and spherical and thus reduces it to the mere endeavour of as many months, as without it would require years to master and even then not understand so thoroughly, and not so permanently apprehend and easier or more likely to forget, as would be the mere word-painting of a landscape or of a human form in the absence of the thing itself or of its picture, paint or photo.

And just here is proof of what the writer has advanced as to

even children manifesting no interest in knowing or learning a thing that has no practical application : A little daughter of mine now five years of age, came to me as she was wont to do time and again when I had drawn for her a house, a tree, a duck, an utensil of any kind. This, the child was always interested in, looking on and waiting patiently till it was done and why : because it was the representative of something in real life.

This time I started at a square which I was subdividing into smaller squares. Not that, said she, supposing I was starting at a tic-tac to in which she had become too old already to take further interest. Not that, she said : no nonsense ; but, wait a bit I said and began to tell her how the little squares were the pattern of the oil-cloth in the passage. Oh yes, she said : now make the marble squares and tiles that I saw in the floor of the new chapel of the basilica, and I cut the corners of the squares and turned them into octagons and squares and other patterns, and oh how delighted she was when I made a lot of hexagons, which looked so pretty to her eye, and how much prettier when I showed her portion of a bees nest and told her how the bees build their little cells in this shape, and how their instinct and God given intelligence manifests itself in using this particular six sided figure, the only one which, while accommodating itself to the body of the insect, fits the one into the other without loss of space, as in the octagon where there are intervening squares, in the square where there are useless corners, and in the circle where there are also useless spaces and of which moreover not only is the periphery greater than that of the hexagon, but where a whole circle would be required for each bee and the required quantity of wax thereby doubled, while, with the figure chosen each component side does double duty.

In conclusion the writer would remark, that as already stated, the Laval Course in his time was and is still in great measure more conducive to clerical requirements or to theological lore to which may be added the legal and medical sciences, than adapted for Engineers, Architects, Scientists Surveyors ; nor have many of those who received their education there become brilliant in the exercise of the latter specialities, and due as stated to an absence or omission of the concrete together with the abstract theories taught ; while of the former, where no mathematics were required, there have been many talented professors and practitioners.

It may also be said as a final allusion to the concreteness of a subject that in treatises on mathematics, and as carried out by the author in his *Plane and Spherical Geom. and Trig.* published in 1866, much valuable space is gained by, instead of giving the enunciation of a proposition in the abstract and then repeating it in the concrete, so writing the enunciation with the letters of reference in the text, but in darker or differently coloured or formed type, that it may be read first without the letters referring to the concrete example giving, and then with them.